9. Další goniometrické vzorce

① SOUČTOVÉ VZORCE

$\forall x, y \in R$:

$\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y$

$\sin(x-y) = \sin x \cdot \cos y - \cos x \cdot \sin y$

$\cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y$

$\cos(x-y) = \cos x \cdot \cos y + \sin x \cdot \sin y$

důkazy: viz učebnici

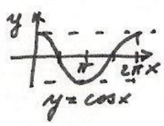
Příklady

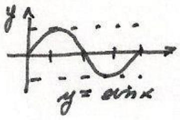
① dokažte, že pro $\forall x \in R$ platí

a) $\underbrace{\cos x}_{L} = \underbrace{\sin\left(x+\frac{\pi}{2}\right)}_{P}$

$P = \sin\left(x+\frac{\pi}{2}\right) = \sin x \cdot \underbrace{\cos\overset{90°}{\frac{\pi}{2}}}_{0} + \cos x \cdot \underbrace{\sin\overset{90°}{\frac{\pi}{2}}}_{1} = \sin x \cdot 0 + \cos x \cdot 1 = \cos x$

$L = \cos x$ $\qquad L = P_{cbd.}$





b) $\sin x = \sin(\pi - x)$

$P = \sin(\pi - x) = \underbrace{\sin\pi}_{0} \cdot \cos x - \underbrace{\cos\pi}_{-1} \cdot \sin x = 0 \cdot \cos x - (-1)\sin x = \sin x$

$L = \sin x$ $\qquad L = P_{cbd.}$

c) $\sqrt{2}\sin\left(x-\frac{\pi}{4}\right) = \sin x - \cos x$

$L = \sqrt{2}\sin\left(x-\frac{\pi}{4}\right) = \sqrt{2}\left(\sin x \underbrace{\cos\overset{45°}{\frac{\pi}{4}}}_{\frac{\sqrt{2}}{2}} - \cos x \cdot \underbrace{\sin\overset{45°}{\frac{\pi}{4}}}_{\frac{\sqrt{2}}{2}}\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x\right) =$

$= \sqrt{2} \cdot \frac{\sqrt{2}}{2}(\sin x - \cos x) = \sin x - \cos x = P_{cbd.}$

d) $\underbrace{\cos x}_{L} = \underbrace{-\cos(\pi - x)}_{P_1} = \underbrace{-\cos(\pi + x)}_{P_2} = \underbrace{\cos(2\pi - x)}_{P_3}$

$P_1 = -\cos(\pi - x) = -\left(\underbrace{\cos\pi}_{-1}\cos x + \underbrace{\sin\pi}_{0}\sin x\right) = -(-\cos x + 0) = \cos x = L$

zú $P_2 = -\cos(\pi + x) = -\left(\underbrace{\cos\pi}_{-1}\cos x - \underbrace{\sin\pi}_{0}\sin x\right) = -(-\cos x - 0) = \cos x = L$

zú $P_3 = \cos(2\pi - x) = \underbrace{\cos 2\pi}_{1}\cos x + \underbrace{\sin 2\pi}_{0}\sin x) = \cos x + 0 = \cos x = L$

② Zjednodušte pomocí součtových vzorců

zú a) $\cos\left(\frac{\pi}{6} - x\right) - \cos\left(\frac{\pi}{6} + x\right) = \cancel{\cos\frac{\pi}{6}\cos x} + \sin\frac{\pi}{6}\sin x - \left(\cancel{\cos\frac{\pi}{6}\cos x} - \sin\frac{\pi}{6}\sin x\right)$

$= 2\underbrace{\sin\frac{\pi}{6}}_{\sin 30° = \frac{1}{2}} \cdot \cos x = 2 \cdot \frac{1}{2}\cos x = \underline{\cos x}$

b) $\sin\left(\frac{\pi}{4} + x\right) - \sin\left(\frac{\pi}{4} - x\right) = \cancel{\sin\frac{\pi}{4}\cos x} + \cos\frac{\pi}{4}\sin x - \left(\cancel{\sin\frac{\pi}{4}\cos x} - \cos\frac{\pi}{4}\sin x\right) =$

zú $= 2\underbrace{\cos\frac{\pi}{4}}_{\cos 45° = \frac{\sqrt{2}}{2}}\sin x = 2 \cdot \frac{\sqrt{2}}{2}\sin x = \underline{\sqrt{2}\sin x}$

③ Vypočítejte hodnoty gon. funkcí bez použití tabulek nebo kalkulačky

a) $x = 105°$ $(105° = 60° + 45°)$

$$\sin 105° = \sin(60°+45°) = \underbrace{\sin 60°}_{\frac{\sqrt{3}}{2}}\underbrace{\cos 45°}_{\frac{\sqrt{2}}{2}} + \underbrace{\cos 60°}_{\frac{1}{2}}\underbrace{\sin 45°}_{\frac{\sqrt{2}}{2}} = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2} =$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\cos 105° = \cos(60°+45°) = \underbrace{\cos 60°}_{\frac{1}{2}}\underbrace{\cos 45°}_{\frac{\sqrt{2}}{2}} - \underbrace{\sin 60°}_{\frac{\sqrt{3}}{2}}\underbrace{\sin 45°}_{\frac{\sqrt{2}}{2}} = \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} =$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

$$\operatorname{tg} 105° = \frac{\sin 105°}{\cos 105°} = \left(\frac{\frac{\sqrt{6}+\sqrt{2}}{4}}{\frac{\sqrt{2}-\sqrt{6}}{4}}\right) = \underbrace{\frac{\sqrt{6}+\sqrt{2}}{\sqrt{2}-\sqrt{6}}\cdot\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}+\sqrt{6}}}_{\text{usměrníme}} = \frac{\overbrace{6 + 2\sqrt{6}\sqrt{2} + 2}^{a^2+2ab+b^2}}{\underbrace{2-6}_{a^2-b^2}} =$$

$(\sqrt{6}+\sqrt{2})^2$ ~ $\sqrt{6}+\sqrt{2}$

$$= \frac{8+2\sqrt{12}}{-4} = \frac{8+4\sqrt{3}}{-4} = \frac{-4(-2-\sqrt{3})}{-4} = -2-\sqrt{3}$$

$$\operatorname{cotg} 105° = \frac{1}{\operatorname{tg} 105°} = \frac{1}{-2-\sqrt{3}} = \frac{-1}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{-2+\sqrt{3}}{4-3} = \sqrt{3}-2$$

b) $x = 15°$ $(15° = 45° - 30°)$

DÚ

$$\sin 15° = \sin(45°-30°) = \underbrace{\sin 45°}_{\frac{\sqrt{2}}{2}}\underbrace{\cos 30°}_{\frac{\sqrt{3}}{2}} - \underbrace{\cos 45°}_{\frac{\sqrt{2}}{2}}\underbrace{\sin 30°}_{\frac{1}{2}} = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} =$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos 15° = \cos(45°-30°) = \underbrace{\cos 45°}_{\frac{\sqrt{2}}{2}}\underbrace{\cos 30°}_{\frac{\sqrt{3}}{2}} + \underbrace{\sin 45°}_{\frac{\sqrt{2}}{2}}\underbrace{\sin 30°}_{\frac{1}{2}} = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} =$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\operatorname{tg} 15° = \frac{\sin 15°}{\cos 15°} = \frac{\frac{\sqrt{6}-\sqrt{2}}{4}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \underbrace{\frac{\sqrt{6}-\sqrt{2}}{\underset{(a+b)}{\sqrt{6}+\sqrt{2}}}\cdot\frac{\sqrt{6}-\sqrt{2}}{\underset{(a-b)}{\sqrt{6}-\sqrt{2}}}}_{\text{usměrníme}} = \frac{\overbrace{(\sqrt{6}-\sqrt{2})^2}^{(a-b)^2}}{\underbrace{6-2}_{a^2-b^2}} = \frac{6-2\sqrt{6}\sqrt{2}+2}{4} =$$

$$= \frac{8-2\sqrt{12}}{4} = \frac{8-4\sqrt{3}}{4} = \frac{4(2-\sqrt{3})}{4} = 2-\sqrt{3}$$

$$\operatorname{cotg} 15° = \frac{\cos 15°}{\sin 15°} = \frac{1}{\operatorname{tg} 15°} = \underbrace{\frac{1}{\underset{(a-b)}{2-\sqrt{3}}}\cdot\frac{2+\sqrt{3}}{\underset{(a+b)}{2+\sqrt{3}}}}_{\text{usm.}} = \frac{2+\sqrt{3}}{\underset{a^2\ \ b^2}{4-3}} = \frac{2+\sqrt{3}}{1} = 2+\sqrt{3}$$

! ZKONTROLUJTE NA KALKULAČCE – DOM. ÚKOL !

④ Dokažte

a) $\underbrace{\sin 2x}_{L} = \underbrace{2\sin x\cos x}_{P}$ $\qquad$ $\underbrace{\cos 2x}_{L} = \underbrace{\cos^2 x - \sin^2 x}_{P}$

$L = \sin 2x = \sin(x+x) = \sin x\cdot\cos x + \cos x\cdot\sin x = 2\sin x\cos x = P$

$L = \cos 2x = \cos(x+x) = \cos x\cdot\cos x - \sin x\cdot\sin x = \cos^2 x - \sin^2 x = P$

b) $\underbrace{\sin 3x}_{L} = \underbrace{3\sin x - 4\sin^3 x}_{P}$ $\qquad$ $\underbrace{\cos 3x}_{L} = \underbrace{4\cos^3 x - 3\cos x}_{P}$

$L = \sin 3x = \sin(2x+x) = \sin 2x\cos x + \cos 2x\cdot\sin x =$

$= 2\sin x\cos x\cos x + (\cos^2 x - \sin^2 x)\sin x =$

$= 2\sin x\cdot\cos^2 x + \cos^2 x\cdot\sin x - \sin^3 x =$

$= 2\sin x(1-\sin^2 x) + (1-\sin^2 x)\sin x - \sin^3 x =$

$= 2\sin x - 2\sin^3 x + \sin x - \sin^3 x - \sin^3 x = 3\sin x - 4\sin^3 x = P$

DÚ $L = \cos 3x = \cos(2x+x) = \cos 2x\cos x - \sin 2x\sin x =$

$= (\cos^2 x - \sin^2 x)\cos x - 2\sin x\cos x\sin x = \cos^3 x - \sin^2 x\cos x - 2\sin^2 x\cos$

$= \cos^3 x - (1-\cos^2 x)\cos x - 2(1-\cos^2 x)\cos x =$

$= \cos^3 x - \cos x + \cos^3 x - 2\cos x + 2\cos^3 x = 4\cos^3 x - 3\cos x = P$

[pozn. – vzorce lze odvodit pomocí Moivreovy věty – viz 3. roč. komplexní č.]

② $\forall x,y\in R$:

$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

$\sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$

$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

– vzorce využíváme při úpravách na součin, nebo naopak na součet

Příklady

⑤ Vyjádřete jako součin

a) $\sin 3y + \sin y = 2\sin\frac{3y+y}{2}\cos\frac{3y-y}{2} = 2\sin 2y\cdot\cos y$

$\left(= 2\cdot 2\underbrace{\sin y\cos y}_{\sin 2y}\cdot\cos y = 4\sin y\cos^2 y\right)$

b) $\cos\frac{\pi}{12} - \cos\frac{\pi}{18} = -2\sin\frac{\frac{\pi}{12}+\frac{\pi}{18}}{2}\sin\frac{\frac{\pi}{12}-\frac{\pi}{18}}{2} = -2\sin\frac{\frac{3\pi+2\pi}{36}}{2}\sin\frac{\frac{3\pi-2\pi}{36}}{2} =$

DÚ

$= -2\sin\frac{5\pi}{72}\sin\frac{\pi}{72}$

c) $\sin x + 0{,}5 = \sin x + \sin\frac{\pi}{6} = 2\sin\frac{\frac{x+\frac{\pi}{6}}{2}}{}\cos\frac{x-\frac{\pi}{6}}{2} = \underline{2\sin\frac{6x+\pi}{12}\cos\frac{6x-\pi}{12}}$

$[0{,}5 = \sin 30° = \sin\frac{\pi}{6}]$

d) $\cos x - 0{,}5 = \cos x - \cos\frac{\pi}{3} = -2\sin\frac{x+\frac{\pi}{3}}{2}\sin\frac{x-\frac{\pi}{3}}{2} = \underline{-2\sin\frac{3x+\pi}{6}\sin\frac{3x-\pi}{6}}$

DÚ

$[\frac{1}{2} = \cos 60° = \cos\frac{\pi}{3}]$

⑥ Dokažte

a) $\underbrace{\mathrm{tg}^2\frac{x}{2}}_{L} = \underbrace{\frac{1-\cos x}{1+\cos x}}_{P}$

$$P = \frac{1-\cos x}{1+\cos x} = \frac{\cos 0 - \cos x}{\cos 0 + \cos x} = \frac{-2\sin\frac{0+x}{2}\sin\frac{0-x}{2}}{2\cos\frac{0+x}{2}\cos\frac{0-x}{2}} = \frac{-\sin\frac{x}{2}\sin(-\frac{x}{2})}{\cos\frac{x}{2}\cos(-\frac{x}{2})} =$$

$$= \frac{-\sin\frac{x}{2}\cdot(-\sin\frac{x}{2})}{\cos\frac{x}{2}\cdot\cos\frac{x}{2}} = \frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}} = \underline{\mathrm{tg}^2\frac{x}{2}} = L \qquad \left[\begin{array}{l}\cos x + 1 \neq 0\\ \cos x \neq -1\\ x \neq \pi + 2k\pi\\ x \neq \pi(2k+1)\end{array}\right]$$

DÚ b) $\mathrm{cotg}^2\frac{x}{2} = \frac{1+\cos x}{1-\cos x}$

$$P = \frac{1+\cos x}{1-\cos x} = \frac{\cos 0 + \cos x}{\cos 0 - \cos x} = \frac{2\cos\frac{0+x}{2}\cos\frac{0-x}{2}}{-2\sin\frac{0+x}{2}\sin\frac{0-x}{2}} = \frac{\cos\frac{x}{2}\cdot\cos(-\frac{x}{2})}{-\sin\frac{x}{2}\sin(-\frac{x}{2})} =$$

$$= \frac{\cos\frac{x}{2}\cdot\cos\frac{x}{2}}{-\sin\frac{x}{2}\cdot(-\sin\frac{x}{2})} = \frac{\cos^2\frac{x}{2}}{\sin^2\frac{x}{2}} = \mathrm{cotg}^2\frac{x}{2} = L \qquad \left[\begin{array}{l}\cos x \neq 1\\ x \neq 0 + 2k\pi\\ x \neq 2k\pi\end{array}\right]$$

- po odmocnění plynou další vzorce - viz dále

⑦ Dokažte, že pro přípustné hodnoty $x, y \in R$ platí

a) $\mathrm{tg}(x+y) = \frac{\mathrm{tg}x + \mathrm{tg}y}{1 - \mathrm{tg}x\,\mathrm{tg}y}$

$$P = \underline{\frac{\mathrm{tg}x + \mathrm{tg}y}{1-\mathrm{tg}x\,\mathrm{tg}y}} = \frac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{1 - \frac{\sin x}{\cos x}\cdot\frac{\sin y}{\cos y}} = \frac{\frac{\sin x\cos y + \cos x\sin y}{\cos x\cos y}}{\frac{\cos x\cos y - \sin x\sin y}{\cos x\cos y}} =$$

$$= \frac{\sin(x+y)}{\cos(x+y)} = \underline{\mathrm{tg}(x+y)} = L$$

b) $\mathrm{cotg}(x-y) = \frac{\mathrm{cotg}x\,\mathrm{cotg}y + 1}{\mathrm{cotg}y - \mathrm{cotg}x}$

DÚ

$$P = \frac{\frac{\cos x}{\sin x}\cdot\frac{\cos y}{\sin y} + 1}{\frac{\cos y}{\sin y} - \frac{\cos x}{\sin x}} = \frac{\frac{\cos x\cos y + \sin x\sin y}{\sin x\sin y}}{\frac{\sin x\cos y - \cos x\sin y}{\sin x\sin y}} = \frac{\cos(x-y)}{\sin(x-y)} = \underline{\mathrm{cotg}(x-y)} = L$$

c) $\mathrm{cotg}(x+y) = \frac{\mathrm{cotg}x\,\mathrm{cotg}y - 1}{\mathrm{cotg}x + \mathrm{cotg}y}$

DÚ

$$P = \frac{\frac{\cos x}{\sin x}\cdot\frac{\cos y}{\sin y} - 1}{\frac{\cos x}{\sin x} + \frac{\cos y}{\sin y}} = \frac{\frac{\cos x\cos y - \sin x\sin y}{\sin x\sin y}}{\frac{\cos x\sin y + \sin x\cos y}{\sin x\sin y}} = \frac{\cos(x+y)}{\sin(x+y)} = \mathrm{cotg}(x+y) = P$$

d) $\operatorname{tg}(x-y) = \dfrac{\operatorname{tg}x - \operatorname{tg}y}{1+\operatorname{tg}x\cdot\operatorname{tg}y}$

Ř:

$$P = \frac{\operatorname{tg}x-\operatorname{tg}y}{1+\operatorname{tg}x\cdot\operatorname{tg}y} = \frac{\frac{\sin x}{\cos x}-\frac{\sin y}{\cos y}}{1+\frac{\sin x}{\cos x}\cdot\frac{\sin y}{\cos y}} = \frac{\frac{\sin x\cos y-\sin y\cos x}{\cancel{\cos x\cos y}}}{\frac{\cos x\cos y+\sin x\sin y}{\cancel{\cos x\cos y}}} = \frac{\sin(x-y)}{\cos(x-y)} =$$

$$= \operatorname{tg}(x-y) = L$$

3 VZORCE PRO POLOVIČNÍ ÚHEL

$$\left|\sin\frac{x}{2}\right| = \sqrt{\frac{1-\cos x}{2}} \qquad \left|\operatorname{tg}\frac{x}{2}\right| = \sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$\left|\cos\frac{x}{2}\right| = \sqrt{\frac{1+\cos x}{2}} \qquad \left|\operatorname{cotg}\frac{x}{2}\right| = \sqrt{\frac{1+\cos x}{1-\cos x}}$$

Příklady

8 Odvoďte vzorce pro poloviční úhel

- vyjdeme z dvojnás. úhlu – $\cos 2x$

$\cos 2x = \cos^2 x - \sin^2 x$ $[\sin^2 x + \cos^2 x = 1]$

$\cos 2x = 1 - \sin^2 x - \sin^2 x$

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

$|\sin x| = \sqrt{\dfrac{1-\cos 2x}{2}}$

subst. $x \to \frac{x}{2}$

$\left|\sin\frac{x}{2}\right| = \sqrt{\dfrac{1-\cos x}{2}}$

$\cos 2x = \cos^2 x - (1-\cos^2 x)$

$\cos 2x = 2\cos^2 x - 1$

$2\cos^2 x = \cos 2x + 1$

$\cos^2 x = \dfrac{1+\cos 2x}{2}$

$|\cos x| = \sqrt{\dfrac{1+\cos 2x}{2}}$

subst. $x \to \frac{x}{2}$

$\left|\cos\frac{x}{2}\right| = \sqrt{\dfrac{1+\cos x}{2}}$

$$\left|\operatorname{tg}\frac{x}{2}\right| = \frac{\left|\sin\frac{x}{2}\right|}{\left|\cos\frac{x}{2}\right|} = \frac{\sqrt{\frac{1-\cos x}{2}}}{\sqrt{\frac{1+\cos x}{2}}} = \sqrt{\frac{\frac{1-\cos x}{\cancel{2}}}{\frac{1+\cos x}{\cancel{2}}}} = \sqrt{\frac{1-\cos x}{1+\cos x}}$$

obd. $\left|\operatorname{cotg}\frac{x}{2}\right|$

9 Vypočítejte $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, $\operatorname{tg}\frac{x}{2}$, $\operatorname{cotg}\frac{x}{2}$, jestliže

$\sin x = \frac{1}{3}$ a $\cos x < 0$

⊕ I., II. ⊖ II., III. => II. kv.

$\cos x$: $\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$|\cos x| = \sqrt{1-\left(\frac{1}{3}\right)^2} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

II. kv. ⊖

$\cos x = -\frac{2\sqrt{2}}{3}$

$$\left|\sin\frac{x}{2}\right| = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{2\sqrt{2}}{3}}{2}} = \sqrt{\frac{3+2\sqrt{2}}{6}}$$

! x… II. kv.

$\frac{x}{2}$ => I. kv. => $\sin\frac{x}{2}$ ⊕ => $\sin\frac{x}{2} = \sqrt{\frac{3+2\sqrt{2}}{6}}$

$$\left|\cos\frac{x}{2}\right| = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\frac{2\sqrt{2}}{3}}{2}} = \sqrt{\frac{3-2\sqrt{2}}{6}}$$

! x… II. kv.

$\frac{x}{2}$ => I. kv. => $\cos\frac{x}{2}$ ⊕ $\cos\frac{x}{2} = \sqrt{\frac{3-2\sqrt{2}}{6}}$

DÚ

$$\left|\operatorname{tg}\frac{x}{2}\right| = \frac{\left|\sin\frac{x}{2}\right|}{\left|\cos\frac{x}{2}\right|} = \sqrt{\frac{\frac{3+2\sqrt{2}}{6}}{\frac{3-2\sqrt{2}}{6}}} = \sqrt{\frac{3+2\sqrt{2}}{3-2\sqrt{2}}}$$

$\frac{x}{2} \to$ I. kv. ⊕

$\operatorname{tg}\frac{x}{2} = \sqrt{\frac{3+2\sqrt{2}}{3-2\sqrt{2}}}$

$\left|\operatorname{cotg}\frac{x}{2}\right| = \dfrac{1}{\left|\operatorname{tg}\frac{x}{2}\right|}$

x… II. kv. => $\frac{x}{2}$… I. kv. ⊕

$\operatorname{cotg}\frac{x}{2} = \sqrt{\frac{3-2\sqrt{2}}{3+2\sqrt{2}}}$

⑩ Dokažte

a) $tg\,2x = \dfrac{2\,tg\,x}{1-tg^2x}$

1 ve vhod. tvaru

$$L = tg\,2x = \frac{\sin 2x}{\cos 2x} = \frac{2\sin x\cos x}{\cos^2x-\sin^2x} : \frac{\cos^2x}{\cos^2x} = \frac{2\,\frac{\sin x}{\cos x}}{1-\frac{\sin^2x}{\cos^2x}} = \frac{2\,tg\,x}{1-tg^2x} = P$$

(lze dokazovat i obráceně – $P \to L$ – jednodušší – viz b))

b) $cotg\,2x = \dfrac{cotg^2x-1}{2\,cotg\,x}$

DÚ

$$P = \frac{cotg^2x-1}{2\,cotg\,x} = \frac{\frac{\cos^2x}{\sin^2x}-1}{2\,\frac{\cos x}{\sin x}} = \frac{\frac{\cos^2x-\sin^2x}{\sin^2x}}{\frac{2\cos x}{\sin x}} = \frac{(\cos^2x-\sin^2x)\cancel{\sin x}}{2\cos x\cdot\sin^2x} =$$

$$= \frac{\cos^2x-\sin^2x}{2\sin x\cos x} = \frac{\cos 2x}{\sin 2x} = cotg\,2x$$

④ DALŠÍ VZORCE

– pro dvojnásobné $tg\,2x$, $cotg\,2x$

$$tg\,2x = \frac{2\,tg\,x}{1-tg^2x} \qquad cotg\,2x = \frac{cotg^2x-1}{2\,cotg\,x}$$

– součtové pro $tg(x\pm y)$, $cotg(x\pm y)$

– viz př. ⑦

$$tg(x\pm y) = \frac{tg\,x \pm tg\,y}{1\mp tg\,x\,tg\,y} \qquad cotg(x\pm y) = \frac{cotg\,x\,cotg\,y\mp 1}{cotg\,y \pm cotg\,x}$$